

# BFKL NLL description of forward jets at HERA

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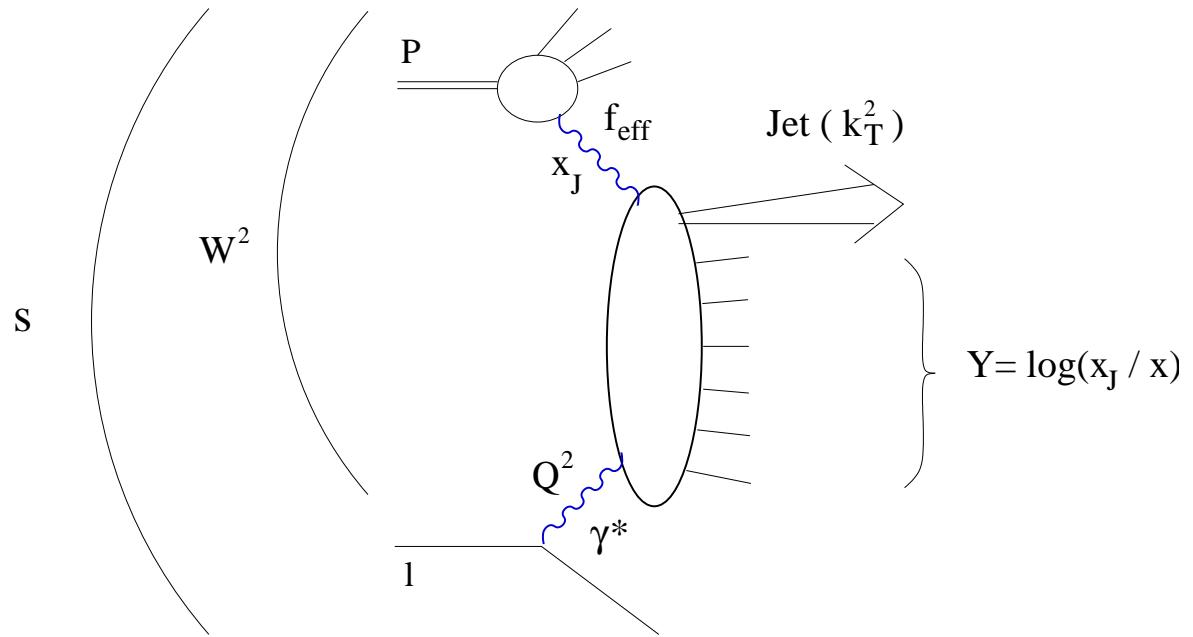
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Contents:

- BFKL-NLL formalism
- Saddle point approximation for the forward jet cross section
- Fit to  $d\sigma/dx$  data
- Comparison with other H1/ZEUS measurements

Work done in collaboration with O. Kepka, C. Marquet, R. Peschanski

## BFKL LO formalism



- Typical kinematical domain where BFKL effects are supposed to appear with respect to DGLAP:  $k_T^2 \sim Q^2$ , and  $Q^2$  not too large
- LO BFKL forward jet cross section
- Saddle point approximation and fits to the H1  $d\sigma/dx$  data: 2 parameters,  $\alpha_S$  in exponential (constant and fitted at LO), and normalisation

## BFKL LO formalism

- BFKL LO forward jet cross section, saddle point approximation:

$$\frac{d\sigma}{dx dk_T dQ^2 dx_{jet}} = N \sqrt{\frac{Q^2}{k_T^2}} \alpha_S(k_T^2) \alpha_S(Q^2) \sqrt{aa} \\ \exp\left(4\alpha(\log 2) \frac{N_C}{\pi} \log\left(\frac{x_J}{x}\right)\right) \\ \exp\left(-aa \log^2\left(\sqrt{\frac{Q}{k_T}}\right)\right)$$

where

$$\frac{1}{aa} = \frac{7\zeta(3)}{\pi} \alpha \log \frac{x_J}{x}$$

- 2 parameters in fits to  $d\sigma/dx$ :  $N$ ,  $\alpha$

## **One parenthesis: cross section calculation**

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- **Two difficulties:** We need to integrate over the bin in  $Q^2$ ,  $x_{jet}$ ,  $k_T$  to compare with the experimental measurement and we need to take into account the experimental cuts (as an example:  $E_e > 10$  GeV,  $k_T > 3.5$  GeV,  $7 \leq \theta_J \leq 20$  degrees....)
- **We perform the integration numerically:** we chose the variables for which the cross section is as flat as possible to avoid numerical difficulties in precision:  $k_T^2/Q^2$ ,  $1/Q^2$ ,  $\log 1/x_{jet}$
- **We take into account some of the cuts at the integration level ( $k_T$  for instance) and the other ones using a toy Monte Carlo**

## How to go to BFKL-NLL formalism?

- Simple idea: Keep the saddle point approximation, and use the BFKL NLO kernel
- Formula at NLL:

$$\frac{d\sigma}{dx} = N \left( \frac{Q^2}{k_T^2} \right)^{\text{exp}} \alpha_S(k_T^2) \alpha_S(Q^2) \sqrt{aa} \\ \exp \left( \alpha_S(k_t^2) \frac{N_C}{\pi} \chi(\gamma_C) \log \left( \frac{x_J}{x} \right) \right) \\ \exp \left( -aa \alpha_S(k_T^2) \log^2 \left( \sqrt{\frac{Q}{k_T}} \right) \right)$$

where

$$\frac{1}{aa} = \frac{3\alpha_S(k_T^2)}{4\pi} \log \frac{x_J}{x} \chi''(\gamma_C) \\ \text{exp} = \gamma_C + \frac{\alpha_S(k_T^2) \chi(\gamma_C)}{2}$$

- Only free parameter in the BFKL NLL fit: absolute normalisation

## BFKL NLL and resummation schemes

- **NLO BFKL:** Corrections were found to be large with respect to LO, and lead to unphysical results
- **NLO BFKL kernels need resummation:** to remove additional spurious singularities in  $\gamma$  and  $(1 - \gamma)$
- **NLO BFKL kernel:**

$$\chi_{NLO}(\gamma, \omega) = \chi^{(0)}(\gamma, \omega) + \alpha(\chi_1(\gamma) - \chi_1^{(0)}(\gamma))$$

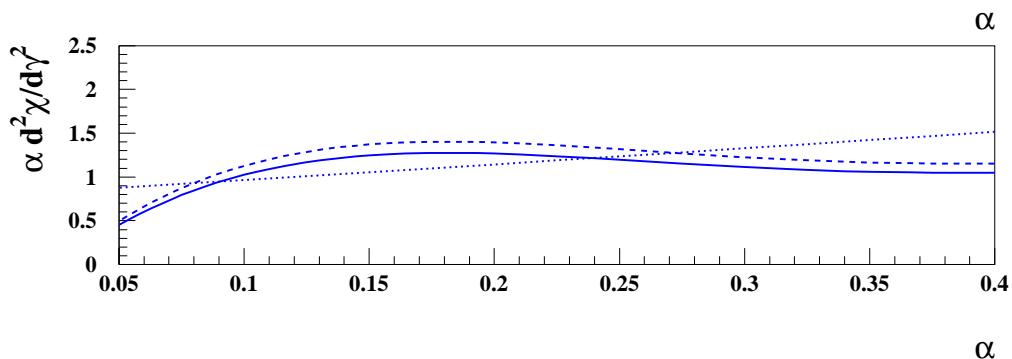
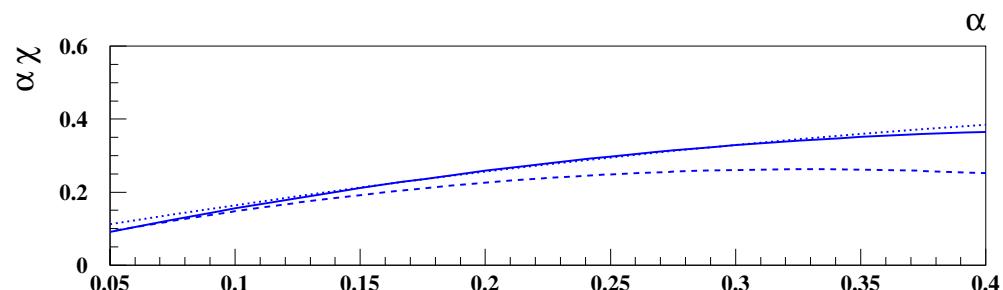
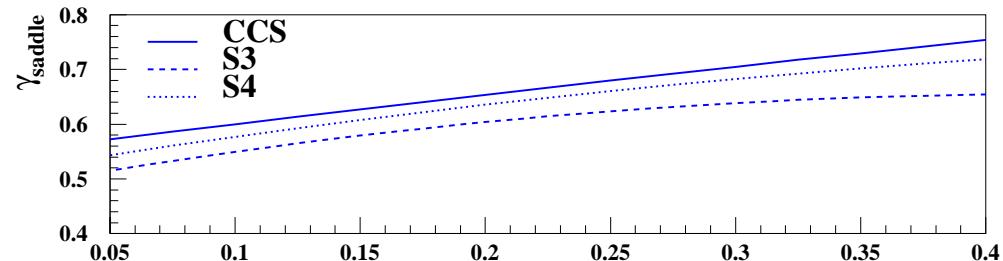
- $\chi_1(\gamma)$ : calculated, NLO BFKL eigenvalues (Lipatov, Fadin, Camici, Ciafaloni)
- $\chi^{(0)}$  and  $\chi_1(0)$ : ambiguity of resummation at higher order than NLO, different ways to remove these singularities, not imposed by BFKL equation, Salam, Ciafaloni, Colferai
- **Transformation of the energy scale:**  $\gamma \rightarrow \gamma - \omega/2$  (Salam) needed for  $F_2$  but not for forward jet cross sections (the problem is symmetric contrary to  $F_2$ )

## **How to determine $\gamma_C$ , $\chi(\gamma_C)$ , and $\chi''(\gamma_C)$ ?**

- **First step:** Knowledge of  $\chi_{NLO}(\gamma, \omega, \alpha)$  from BFKL equation and resummation schemes ( $\omega$  is the Mellin transform of  $Y$ )
- **Second step:** Use implicit equation  $\chi(\gamma, \omega) = \omega/\alpha$  to compute numerically  $\omega$  as a function of  $\gamma$  for different schemes and values of  $\alpha$
- **Third step:** Numerical determination of saddle point values  $\gamma_C$  as a function of  $\alpha$  as well as the values of  $\chi$  and  $\chi''$
- Study performed for three different resummation schemes: S3 and S4 from Gavin Salam, and CCS from Ciafaloni et al.
- For more information: see R. Peschanski, C. Royon, and L. Schoeffel, Nucl.Phys.B716 (2005) 401, [hep-ph/0411338](https://arxiv.org/abs/hep-ph/0411338)

$\gamma_C$ ,  $\chi(\gamma_C)$ , and  $\chi''(\gamma_C)$  as a function of  $\alpha$

Determination of  $\gamma_C$ ,  $\chi(\gamma_C)$ , and  $\chi''(\gamma_C)$  as a function of  $\alpha$



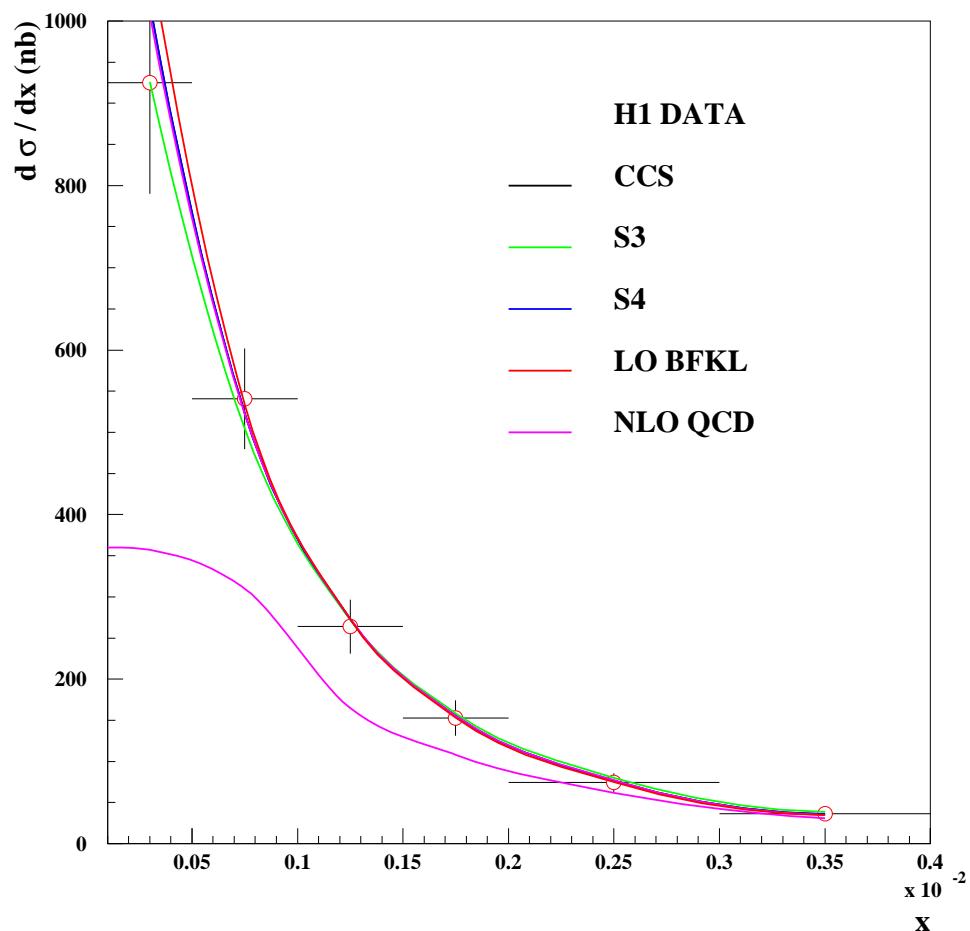
## Fit procedure

- Fit to H1  $d\sigma/dx$  data only
- Fit using the 6 data points or 5 points only, removing the lowest  $x$  point
- $\alpha$  (constant) is found to be small at LO, of the order of 0.1, and  $\alpha_S(k_T^2)$  is imposed using the renormalisation group equation at NLL

fit	data set	$\chi^2/dof$	$N$	$\alpha$
LO	6 pts	13/4 (0.47)	0.42	0.102
LO	5 pts	2.4/3 (0.15)	0.37	0.133
CCS	6 pts	22.0/5 (0.6)	0.91	-
CSS	5 pts	2.4/4 (0.21)	0.95	-

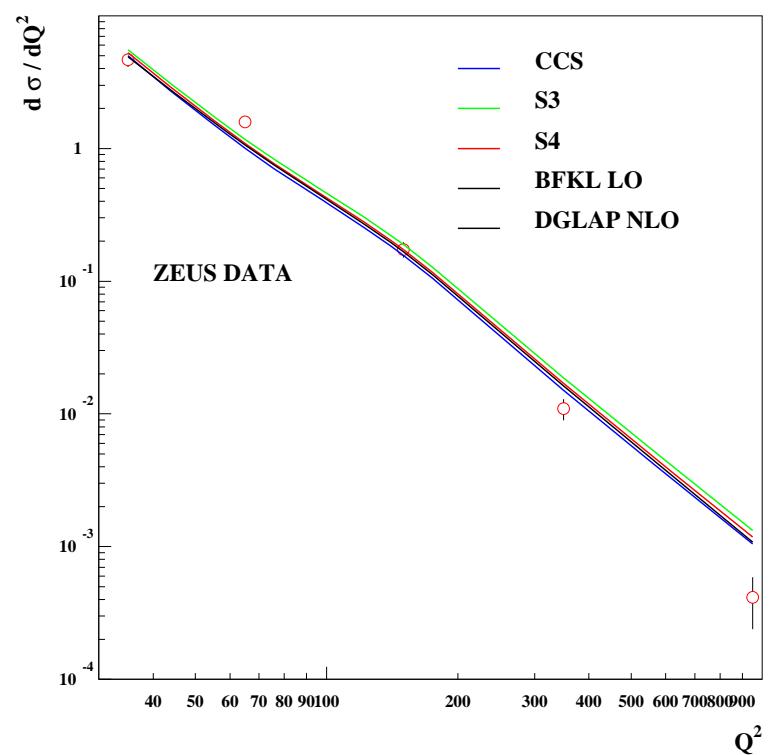
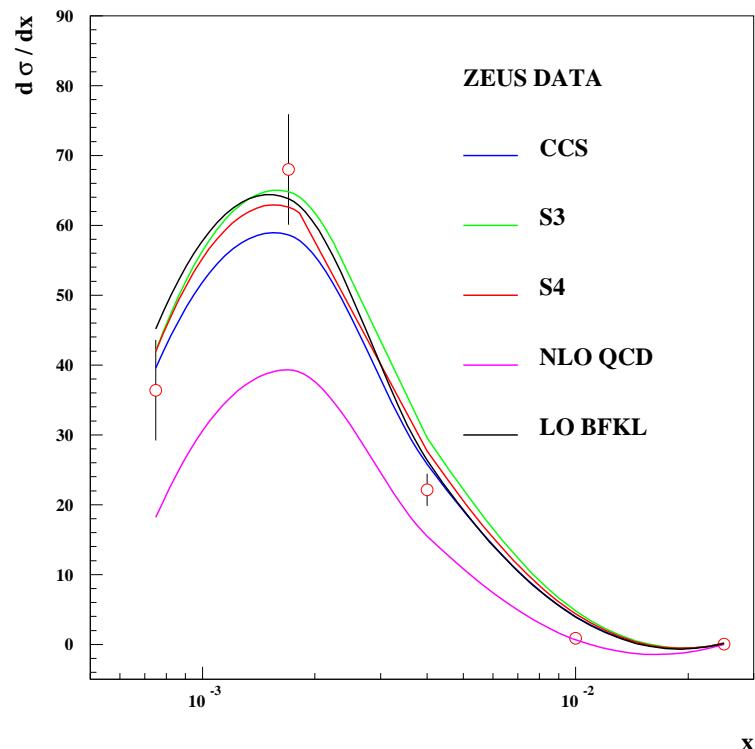
## Fit results

- $\chi^2$  for CCS: 2.4 (0.2), S3: 15.5 (0.8), S4: 4.2 (0.2)
- Good description of H1 data using BFKL LO and BFKL NLL formalism, DGLAP-NLO fails to describe the data
- BFKL higher corrections found to be small (We are in the BFKL-LO region, cut on  $0.5 < kT^2/Q^2 < 5$ )



## Comparison with ZEUS data

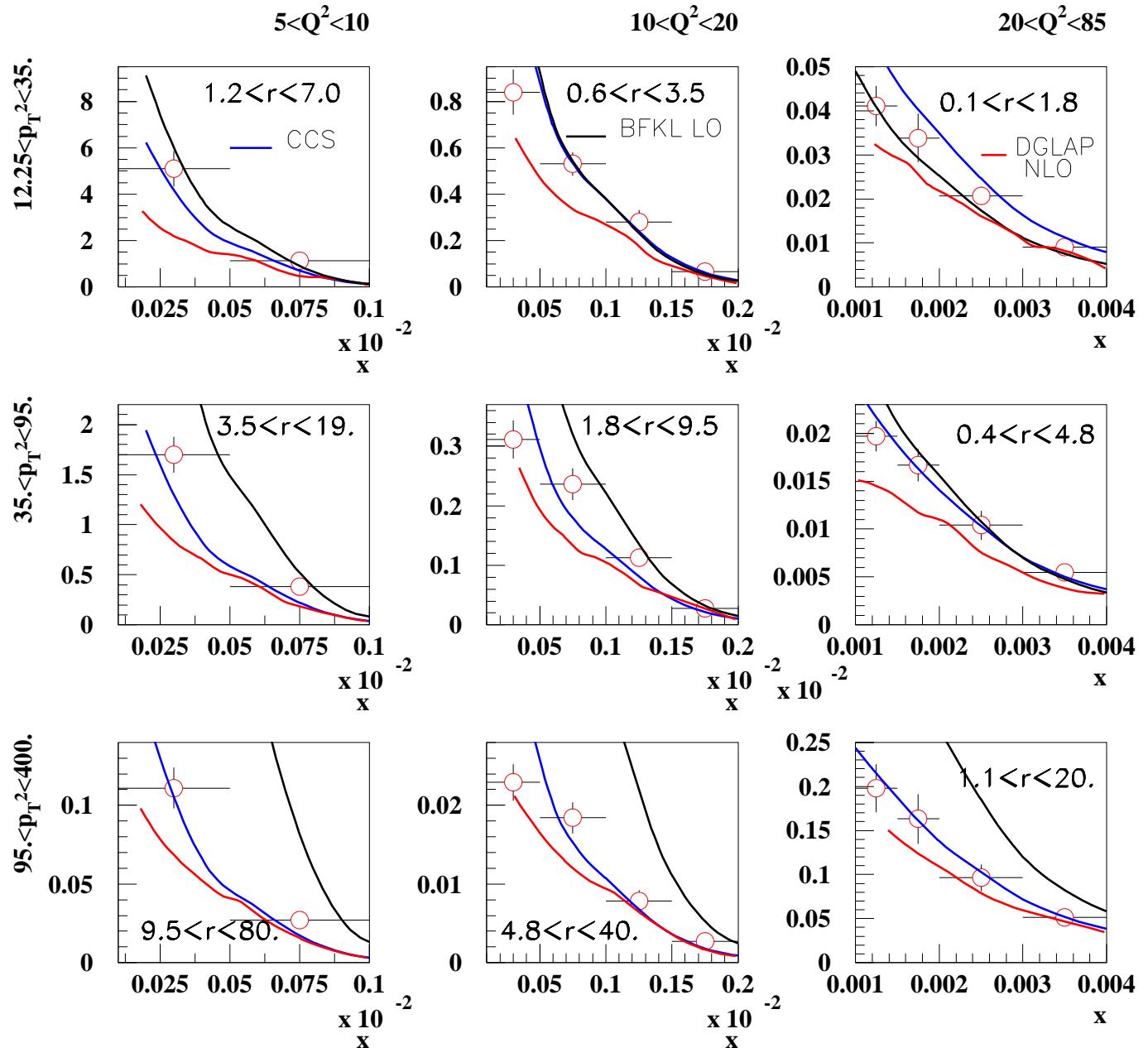
Comparison with ZEUS  $d\sigma/dx$  and  $d\sigma/dQ^2$  data (similar with  $d\sigma/dk_T$  data)



## Comparison with H1 triple differential data

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$d\sigma/dx dp_T^2 dQ^2$  - H1 DATA



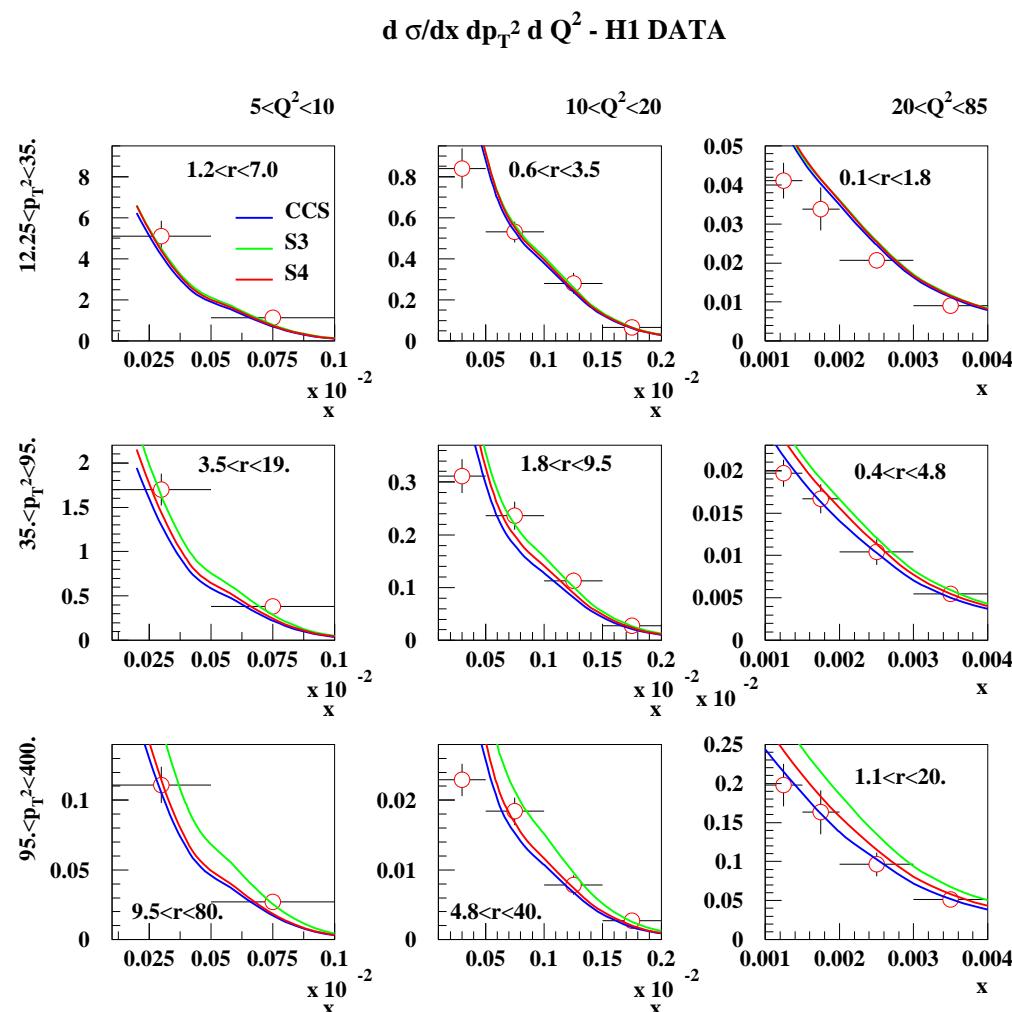
## **Comparison with H1 triple differential data**

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- DGLAP NLO predictions cannot describe H1 data in the full range, and large difference between DGLAP NLO and DGLAP LO results (DGLAP NLO includes part of the small  $x$  resummation effects)
- BFKL LO describes the H1 data when  $r = k_T^2/Q^2$  is close to 1
- BFKL LO fails outside the region  $r \sim 1$  specially at high  $Q^2$
- BFKL higher order corrections found to be small (as expected) when  $r \sim 1$
- Higher order BFKL corrections larger when  $r$  further away from 1, where the BFKL NLL prediction is closer to the DGLAP one ( $Q^2$  resummation effects are starting to be large)
- BFKL NLL gives a good description of data over the full range: first success of BFKL higher order corrections, shows the need of these corrections

## Comparison with H1 triple differential data

- Comparison between the three resummation schemes: CCS, S1 and S3
- CCS and S4 lead to similar description of data while S3 is slightly disfavoured



## Conclusion

- DGLAP NLO fails to describe forward jet data
- First BFKL NLL description of H1 and ZEUS forward jet data: very good description
- The BFKL scale which is used in the exponential  $\alpha_S(k_T^2)$  can describe the H1 cross section measurements
- Higher order corrections small when  $r = kt^2/Q^2 \sim 1$  and larger when  $r$  is further away from 1 as expected
- BFKL NLL formalism leads to a better description than the BFKL LO one for the triple differential cross section: Resummed BFKL NLO kernels include part of the evolution in  $Q^2$